Tav Null: 'Below' the Empty Set, by Nicholas W. Bochenek, first shared 15th June 2025 as part of a virtual library within 'the Fellowship of Infinity' on Steam, revised 28th March 2026

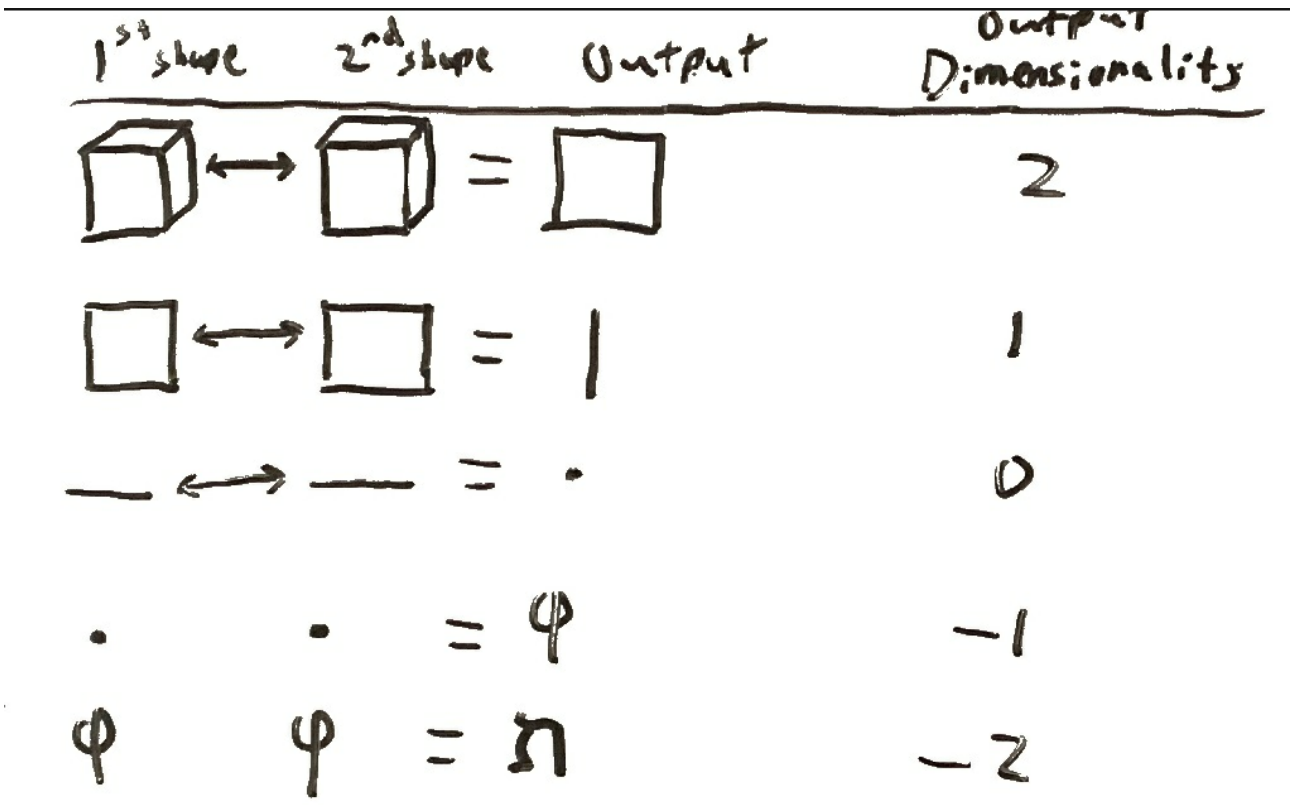
Abstract
We take the reader into an abstract space to uncover an analogy to n dimensional space, iterate it into an analogue of -2 dimensions, and show a way in which it differs severely from +2 dimensional space. We provide a consistent procedure for iterating our abstract shapes that is clearly consistent with known results in the more practical analogues. By creating an entirely new mathematical framework, we are 'free from the burden of applied mathematics and its sensibility' (put that on a mug), so as to streamline our process.

Introduction
The difference between nothing and no thing is not arbitrary. No thing is generally referring to a quantity of zero for a particular object, whereas nothing is nonspecific. A practical example of this difference appears in editing of digital media, such as editing an image. If we begin with an image of no thing, such as a blank, white image, then we can edit this image. Although the editing process may have no effect, such as changing the hue or saturation, the operation is still understood by the computer. If, however, we were to try to apply the same hue or saturation changes but had no image opened whatsoever, then the computer program may give an error, or completely disable the editing options until an image is loaded. The blank image is an analogy for the number zero, and having no images is an analogy for the empty set, which will be described in the next section.
The concept of the differences between zero and the empty set can be extended. We formulate a geometric workspace to consider an analogue of dimensions of space as we intuitively understand and, ultimately, demonstrate what happens if we continue on our 'iteration' from an empty set-like concept into an abstract mathematical object we label Tav null, denoted with the last letter of the Hebrew alphabet, ת.

Preliminaries
Definition 1: the real number line is the set of numbers from negative infinity to positive infinity. It includes zero, irrationals such as the square root of two, integers such as negative three, and transcendentals such as pi, π. It does not include imaginary numbers, amongst many other more abstract things.

Definition 2: a real number range is a subset of the real numbers defined by a lowest and a highest endpoint (minimum and maximum values respectively), along with all real numbers contained between the endpoints. It will be called 'zoordinate range' at times, although will always refer to the range of real numbers being used.

Content Formulation
There is a layman's approach to this idea before we get into the dense formulation. Using the picture on the next page, the reader can begin from the top left, reading across to the right one row at a time, then repeat for the rows descending down the page, as the most common Western reading method would recommend. The first row indicates two cubes which, in contact, touch in an area of a square; the second row indicates two squares which would touch in a straight line of contact; the third row would show a singularity (point) contact between two straight lines; and the fourth and fifth rows become more abstract. A requirement is that the movement of the objects becomes increasingly restricted to enforce this shape iterating, with no movement defined for the singularities on row five, yielding an empty contact zone. The sixth row is yet more poorly defined, and the result is labelled with Tav null. Being poorly defined is a construction mechanism that this paper is boldly plunging into.



For a formal approach, we are going to create some words that are blatantly similar to their analogous counterparts. Maintaining a separation from applied mathematics streamlines the process of describing the abstract concepts present in this paper.

Definition 3: a zoint is a mathematical object of infinitesimal size. These are analogous to points, and they have an analogue to location where applicable, using real numbers.

Definition 4: a zimension is a direction defined by zoints being allocated a single real number. A zoint can be represented in k zimensions with k real numbers, called zoordinates, analogous to coordinates. They have the restriction that two zimensions j and k are different if and only if changing the value of the j zoordinate for a zoint has no effect on the k zoordinate, for any zoint. This is analogous to having two different dimensions that are linearly independent.

Definition 5: the zimensionality of a zoint is the amount of different zimensions a zoint resides in, i.e. the amount of zoordinates required to define the zoint. The plurality of the word zoordinates is important as each real number assigned to a zoint is treated separately from the others. This is related to the idea of 'vector space basis' which we don't cover here. It is important to take note that the zimensionality number will change from a process to be defined later.

Definition 6: zovement is the shifting of zoint(s) along a zimension such that exactly one zoordinate changes quantity equally for all zoints. Zovement is 'defined' for any zimension in which the zube has a non-zero zoint range. This is analogous to moving point(s) in one particular dimension only, including moving a cube without morphing its shape, and the definedness is analogous to not allowing a cube to move in the fourth dimension, a square to move in the third dimension, so on.

Definition 7: a zube is the collection of all zoints that span a fixed zoordinate range across every zimension of its zimensionality. It also has the 'orthogonal' restriction that the zoordinate range when travelling strictly through one zimension will always have the same real number range,

regardless of the starting zoint, so long as it remains within the zube. The analogy here is that an n-dimensional cube (such as a cube or a square) aren't sheared, i.e. the edges or faces are always at right angles to any adjacent ones that exist.
The zimensionality is represented before the word 'zube' with the letter 'n', such as n-zube, or a fixed number if it is known. A 2-zube is a two-zimensional zube, for example, analogous to a 2-dimensional cube, i.e. a square. A 0-zube is a single zoint. Importantly, we don't consider k-zubes that reside in more or less than k zimensions.

Definition 8: the 'start' of an n-zube is the lowest value that the real number of a zoordinate can take without residing outside the n-zube. The 'end' has a similar construction but for highest real number value instead of lowest. The start and the end are analogous to opposing faces of a cube, or opposite edges on a square.

Definition 9: two n-zubes are said to be 'closer' in the j zimension after a zovement in the j zimension if the distance between an end of one n-zube is closer to the start and closer to the end of the other n-zube than before the zovement, where distance is calculated as the difference between zoordinate real numbers. This is to specify that the zubes are approaching each other without an increase of 'overlap' which we will define next.

Definition 10: two n-zubes are said to 'overlap' there is a non-zero zoordinate range contained entirely in both n-zubes for each n zimensions. Analogously in 3D space we distinguish between two 3 dimensional cubes that intersect with a non-zero volume (overlap of a 3 dimensional shape) verses only touching at the surface with no intersecting 3 dimensional volume.

Definition 11, Boundary of Impact: begin with two n-zubes. Also, fix n for both n-zubes and align them such that they are sufficiently separated in one zoordinate (call it the s zimension) to have no zoint overlap but aligned in the other zimensions by having the same starts and ends for all other zimensions. This is analogous to separate chunks of meat aligned on a skewer. Apply zovement to one n-zube along the s zimension to make them closer until the end zoints of one n-zube (in the s zimension) meet the start points of the other n-zube (in the same zimension). At the exact moment that two n-zubes cannot get any closer without overlap, the boundary of impact is the resulting zoints that are contained in both n-zubes.

Proposition 12: the boundary of impact between two n-zubes is an 'n-1'-zube.
The 3D analogy of this claim is that two cubes hitting each other will share a surface. This is ignoring realistic physics and assuming that no space remains between the two cubes. The alignment of them like chunks on a skewer ensures we preserve the n-dimensional cube shape, and get a square instead of a rectangle, triangle, trapezoid, so on.

Proof: the meeting of two n-zubes, end to start with no overlap, indicates that they will share zoints. Due to their alignment, they have the same start and end zoordinates, ignoring the zoordinates of the zimension in which they are separated (call it k). These zoints must have the orthogonal restriction as the zoints from the boundary of impact constitute a complete start/end of an n-zube which itself has the orthogonal restriction. Also, as the zoints span the start to end of each zimension other than k, with no overlap, they form a zube with each zimension except k, meaning they form a zube with zimensionality n-1. Thus, we have an 'n-1'-zube.

It is important to insist on the initial separation between the two n-zubes so as to have no overlap, and to undergo zovement until a minimum separation occurs. This restriction will cause issues that we deliberately want, such as when n=0 and there are no zimensions to have zovement defined in the first place.

Results

The resulting decrease of zimensionality number comes directly from the restriction of no overlapping allowed. For most cases, we still have defined zovement until the moment at which overlap would result, but with the lower zimensional cases we have somewhat degenerative clarity.

Consider the boundary of impact procedure for two 0-zubes, analogous to singularities. The strict rule of no overlapping indicates that they must be distinctively separate, but there are no zimensions defined through which they can move as the zovement is restricted to the number of zimensions as the zube itself (zimensionality). As a result, there are no points that can satisfy this boundary of impact procedure, so we obtain the zube of no zoints, with n=-1. The zube of no zoints is analogous to the empty set, although we haven't defined sets here so we instead call it the 'empty zube'.

Consider the boundary of impact between two empty zubes. Not only are there no zoints to consider, but we are trying to consider definedness for zovement where the zimension number is -1, meaning that our boundary of impact procedure is incapable of producing a zube with any meaningful analogy, being highly degenerate. The result is a zimensionality number of -2, and each successive iteration reduces the zimensionality number by 1. The '-2'-zube we label Tav null. The crux for this procedure gaining degeneracy is from needing defined zovement zimensions and the resulting limitations for the boundary of impact procedure.

Discussion, Extensions and the Need for Peer Review

Some of the formulation here was to compare the empty set with geometric shapes by creating a middle ground for the two.

I assume that a common assumption about -1 dimensional shapes is false; the assumption being that -1 and +1 one are similar for dimensionality, and both consist of analogues of the straight line. The dimensionality number is presumed to me more of just a label and 0-dimensional is not any type of reflex point the way 0 is a reflex point for the real number line (i.e. magnitude loses its meaning when talking about numbering for dimensions). While 0-dimensional space keeps us somewhat confident that we have no directions of movement defined, for '-1'-dimensional movement we somewhat throw our hands up in the air, only to return our hands back down to give it a label. I assume that this approach of labelling the bizarre is acceptable based on similar uses for set theory, such as for labelling larger and larger infinite cardinalities, where they are not the same despite all being infinite, so can thus be differentiated with labels. With that in mind, we could potentially extend this paper by exploring differences between our empty zube and Tav null, if differences are present. Otherwise, an amendment to this paper may be needed. This would be the main priority for peer-reviewing, as concept of Tav null needs to be different to the empty zube for this paper to be providing anything new.

How may this combine with fractional dimensions, i.e. fractals?

The more obvious extension to this paper would be looking at the ongoing analogues that have been present throughout. In topology one may try connecting zimensions to dimensions, zubes to cubes, and the remarkable result that the empty set is still n=-1 both in this analogue (empty zube) and in topology. More to the point, has it already been done in topology? If so, what are the similarities and differences?

Set theory may have something to say, in particular about definition 9, where we define closer zubes from zovement. Set theorists have a variety of closeness-related definitions, including using upper and lower bounds, maximum and minimum elements of a set, and supremum and infimum elements. We avoided such variety, although it may be worthwhile to shift our setup into sets of numbers in order to analyse further.

When creating degenerate zubes (n negative), we rely on the zimensionality number causing issues. For n=0, having nowhere for a zoint to go is acceptable, but having -1 directions to go is, we claim, the location of the degeneracy. However, when using the boundary of impact to iterate the empty zube into Tav null, not only do we have -1 zimensions to use, we also have empty zubes which

contain no zoints, much like trying to grab something that isn't there. Does this cause additional degeneracy? It may be worthwhile investigating exactly what degeneracy is. This may be the way to identify whether Tav null and the empty zube are different or the same.
There may be a connection between boundary of impact for the empty zubes and vacuous truths, in that nothingness can take on any property and it will not contradict anything else, such as the colour of my spaceship being pink which is vacuously true as I do not have a spaceship to begin with.
Another obvious extension may be extending the boundary of impact iterations indefinitely, and creating yet another label for the '-infinity'-zube. Beyond that, combining with infinite set theory may yield interesting, yet mind-boggling results.